/02 Characterising Cycles

Characterising Cycles

COVARIANCE STATIONARY:

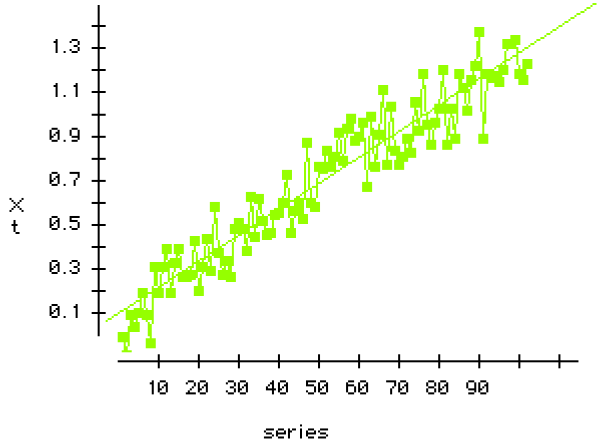
Time Series:

The most basic form of time-series analysis examines trends that are sustained movements in the variable of interest in a specific direction.

Trend analysis often takes one of two forms:

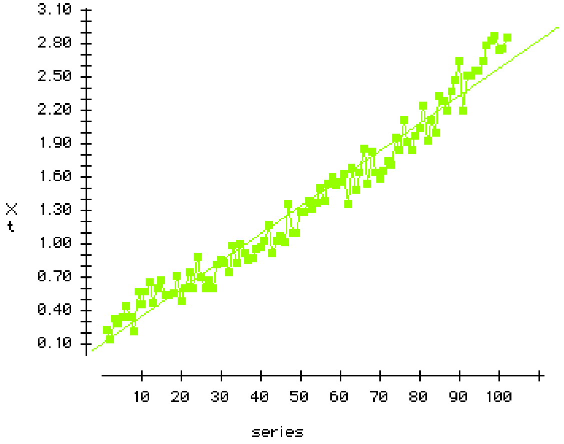
1. Linear trend analysis, in which the dependent variable changes at a constant rate over time.

$$y_t = b_0 + b_1 t + \varepsilon_t$$



2. Log-linear trend analysis, in which the dependent variable changes at an exponential rate over time or constant growth at a particular rate

$$\ln(y_t) = b_0 + b_1 t + \varepsilon_t$$



How do we decide between linear and log-linear trend models?

- Is the estimated relationship persistently above or below the trend line?
- Are the error terms correlated?
- We can diagnose these by examining plots of the trend line, the observed data, and the residuals over time.

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➢ COVARIANCE STATIONARY:

Autoregressive time-series models:
Abbreviated as AR(*p*) models, the *p* indicates how many lagged values of the dependent variable are used and is known as the "order" of the model.
Covariance-stationary series:
A time series is said to be covariance stationary if its mean and variance do not change over time.
Time series that are not covariance stationary have linear regression estimates that are invalid and have no economic meaning.
For a time series to be stationary,

1. The expected value of the series must be finite and constant across time.
2. The variance of the series must be finite and constant across time.
3. The covariance of the time series with itself must be finite and constant for all intervals over all periods across time.

Visually, we can inspect the time-series model for a mean and variance that appear stationary as an initial screen for likely stationarity.

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COVARIANCE STATIONARY:

Residual autocorrelation:

The autocorrelation between one time-series observation and another one at distance *k* in time is known as the *k*th order autocorrelation.

A correctly specified autoregressive model will have residual autocorrelations that do not differ significantly from zero.

Testing procedure:

1. Estimate the AR model and calculate the error terms (residuals).
2. Estimate the autocorrelations for the error terms (residuals).
3. Test to see whether the autocorrelations are statistically different from zero.

   This is a *t*-test, which, if the null hypothesis of no correlation is rejected, mandates modification of the model or data.
   A failure to reject the null indicates that the model is statistically valid.

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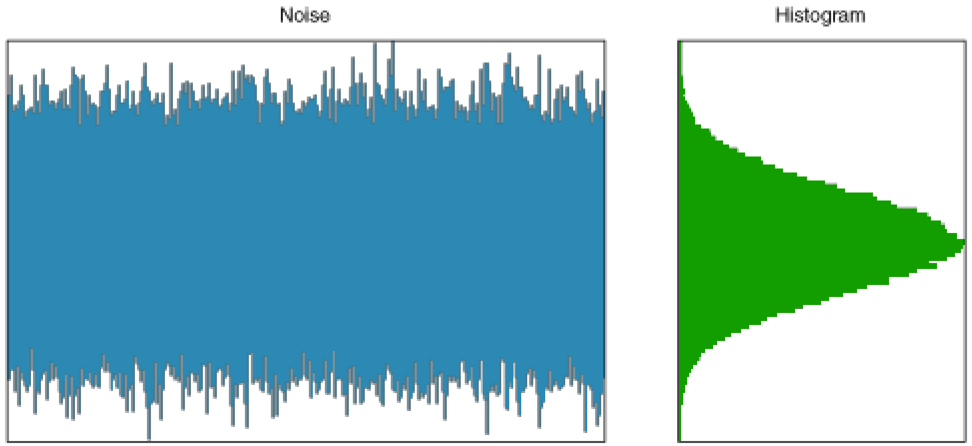
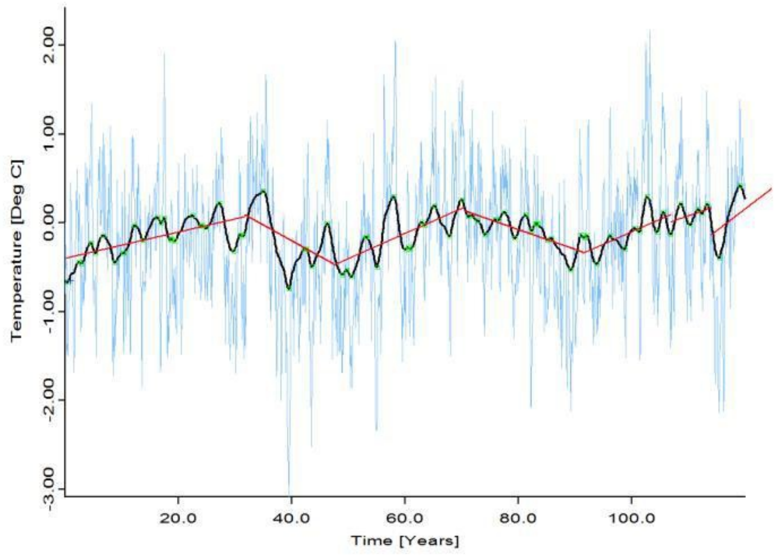
➢ WHITE NOISE:

- **A time series** process with a zero mean, constant variance, and no serial correlation is referred to as a white process (or zero-mean white noise).
- This is the simplest type of time series process and it is used as a fundamental building block for more complex time series processes.
- Even though a white noise process is serially uncorrelated, it may not be serially independent or normally distributed.
- **Independent white noise**: A time series process that exhibits both serial independence and a lack of serial correlation.
- **Normal white noise:** A time series process that exhibits serial independence, is serially uncorrelated and is normally distributed.





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LAG OPERATORS:

- A lag operator quantifies how a time series evolves by lagging a data series.
- It enables a model to express how past data links to the present and how present data links to the future.

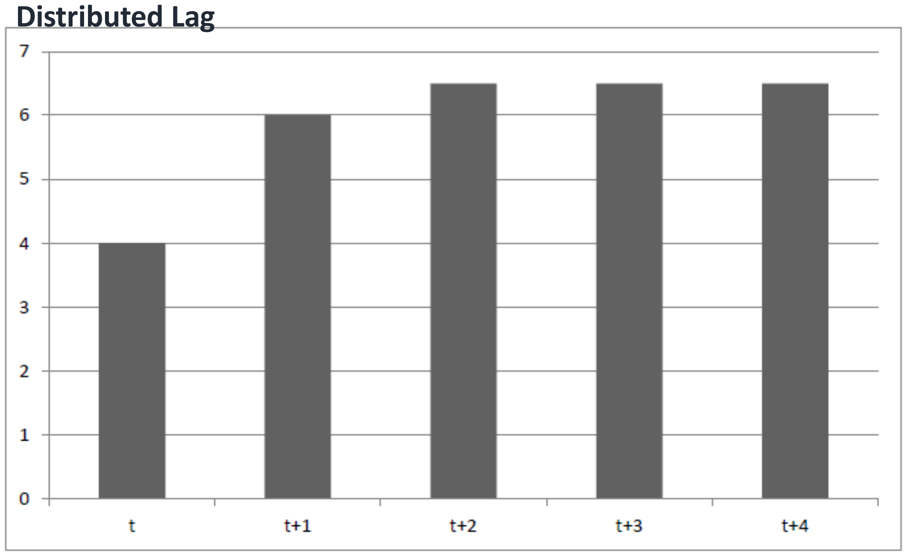
- A lag operator, L satisfies $Ly_t = y_{t-1}$

- A common lag operator is a first-difference operator Δ, which applies a polynomial in the lag operator as follows:
- $\Delta y_t = (1 - L)y_t = y_t - y_{t-1}$

- **Distributed lag**: is a weighted sum of present and past values in a data series, achieved by lagging present values upon past values.

Example:

$L^{-1}y$	y	Ly	L^2y
⋮	⋮	⋮	⋮
2012 → 5.4			
2013 → 5.5	2013 → 5.4		
2014 → 5.8	2014 → 5.5	2014 → 5.4	
⋮	2015 → 5.8	2015 → 5.5	2015 → 5.4
	⋮	2016 → 5.8	2016 → 5.5
		⋮	2017 → 5.8
⋮	⋮		⋮



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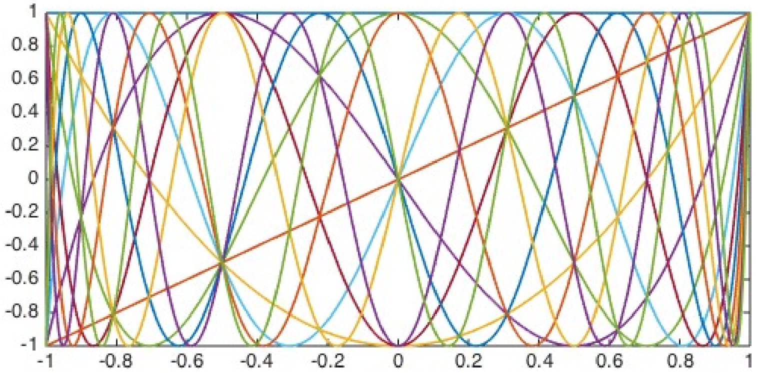
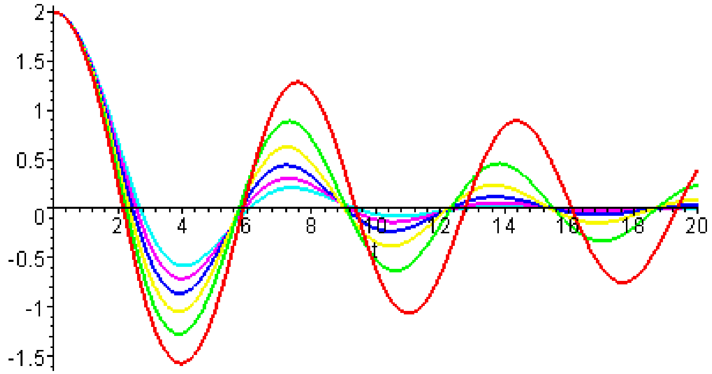
WOLD'S REGRESENTATION THEOREM:

- **Wold's representation theorem** is a model for the covariance stationary residual.
  - The model is constructed after making provisions for trend and seasonal components.
  - The theorem enables the selection of the correct model to evaluate the evaluation of covariance stationary.
  - Wold's representation utilizes an infinite number of distributed lags, where the one-step-ahead forecasted error terms are known as innovations.
- The **general linear process** is a component in the creation of forecasting models in a covariance stationary time series.
  - It uses Wold's representation to express innovations that capture an evolving information set.
  - These evolving information sets move the conditional mean over time.
  - Thus, it can model the dynamics of a times series process that is outside of covariance stationary.
- **Infinite polynomials** that are a ratio of finite-order polynomials are known as **rational polynomials**.
  - The distributed lags constructed from these rational polynomials are known as **rational distributed lags**.
  - With these lags we, can approximate Wold's representation.





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➢ **ESTIMATING THE MEAN AND AUTO-CORRELATION FUNCTIONS:**

❑**Sample Mean:**

$$\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$$

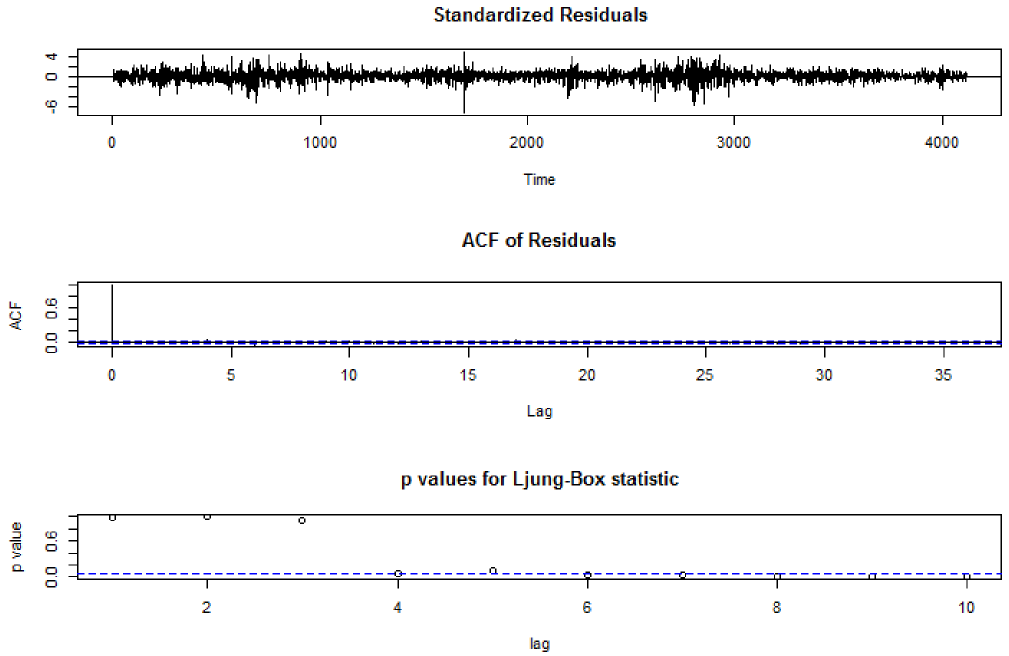
❑**Sample Autocorrelation:** The sample auto-correlation estimates the degree to which white noise characterizes a series of data.

$$\hat{\rho}(\tau) = \frac{\sum_{t=\tau+1}^{T}[(y_t - \bar{y})(y_{t-\tau} - \bar{y})]}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

❑**Q-statistic:** is used to measure the degree to which correlations vary from zero and whether white noise is present in a dataset.

❑**The Box-Pierce Q-statistic:** it reflects the absolute magnitudes of the correlations, because it sums the squared auto-correlations.

❑**The Ljung- Box Q-statistic:** is similar to the Box-Pierce Q-statistic except that it replaces the sum of squared auto-correlations with a weighted sum of squared auto-correlations.



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